Перенормировка вязкости в квантовополевой модели турбулентности на основе вейвлет преобразования

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Abstract

Statistical theory of turbulence in viscid incompressible fluid, described by the Navier-Stokes equation driven by random force, is reformulated in terms of scale-dependent fields $\mathbf{u}_a(x)$, defined as wavelet-coefficients of the velocity field $\mathbf{u}$ taken at point x with the resolution a. Applying quantum field theory approach to the generating functional of random fields $\mathbf{u}_a(x)$, we have shown the velocity field correlators $\langle \mathbf{u}_{a_1}(x_1) \ldots \mathbf{u}_{a_n}(x_n) \rangle$ are finite by construction for the random stirring force acting at prescribed large scale L. Since there are neither UV nor IR divergences, regularization is not required, and renormalization group invariance becomes merely a symmetry that relates velocity fluctuations of different scales. The one-loop corrections to viscosity and to the pair velocity correlator are calculated. This gives deviations from Kolmogorov spectrum.

Subjects

- Quantum field theory approach to hydrodynamic turbulence
- The Kolmogorov theory of turbulence
- Renormalization group
- Separating fluctuations of different scales using wavelet transform
- Quantum field theory of scale-dependent fields $u_a(x, t)$ finite by construction
- Exact renormalization group in multiscale formalism
- Applications to other models

Hydrodynamic turbulence

Turbulence:

Chaotic fluid flow emerging from a laminar flow when certain parameters of the flow exceed critical values

Turbulence is assumed to be

homogeneous $P[\mathbf{u}(t, \mathbf{x} + \Delta\mathbf{x})] = P[\mathbf{u}(t, \mathbf{x})]$

stationary $P[\mathbf{u}(t + \Delta t, \mathbf{x})] = P[\mathbf{u}(t, \mathbf{x})]$

isotropic $P[\mathbf{u}(t, \hat{R}(\theta)\mathbf{x}] = P[\mathbf{u}(t, \mathbf{x})]$

ergodic $\langle f[\mathbf{u}(t, \mathbf{x})]\rangle = \lim_{T\to\infty} \frac{1}{T}\int_0^T f[\mathbf{u}(t, \mathbf{x})]dt$

In many practical cases the fluid flow can be considered as incompressible $\nabla \cdot \mathbf{u} = 0$ and described by the Navier-Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \nu\Delta\mathbf{u} - \nabla p + \mathbf{f}(t, \mathbf{x}, \cdot)$$

driven by random force $\mathbf{f}(t, \mathbf{x}, \cdot)$

The Kolmogorov theory of Turbulence

Studies of turbulence stem from the Reynolds similarity law [O.Reynolds, *Phil. Trans. R. Soc. Lond. A*, **186**(1895)123-164]: Let

$$\tilde{u} = \frac{u}{V}, \quad \tilde{x}_i = \frac{x_i}{L}, \quad \tilde{t} = t\frac{V}{L}, \quad \tilde{P} = \frac{P}{\rho V^2}$$

be dimensionless variables; with V being the macroscopic flow velocity, L being the flow size, than the flows of the same type with equal Reynolds numbers $Re = \frac{VL}{\nu}$ are similar to each other.

Kolmogorov hypotheses (K41) *Dokl.Akad.Nauk SSSR*, 1941, v.**30**, pp. 9-13

I. For the locally isotropic turbulence the n-point velocity field distributions F_n are uniquely determined by kinematic viscosity ν and the energy dissipation rate per unit of mass ε.
II. In the *inertial range*, i.e., if the observation scale l is much bigger than the dissipation length $l_0 = \nu^{3/4}/\varepsilon^{1/4}$, the distributions are uniquely determined by the energy dissipation rate per unit of mass ε, and do not depend on ν.

Taylor hypothesis: $\partial_t f = V_i \nabla_i f$

Structure functions and energy spectrum

Energy spectrum

$$E = \int |u(\mathbf{x})|^2 d^3\mathbf{x} = \int_0^\infty E(k)dk$$

where

$$E(k) = \int_{|\mathbf{k}|=k} \langle u(\mathbf{k})u(-\mathbf{k})\rangle d^3\mathbf{k}$$

$$E(k) = 4\pi\mathbf{k}^2 F(\mathbf{k})$$

According to Kolmogorov dimension counting

$$E(k) = C\varepsilon^{2/3}k^{-5/3}$$

Structure functions

$$S_q(l) = \langle|\mathbf{u}(x) - \mathbf{u}(x+l)|^q\rangle$$

According to Kolmogorov dimension counting

$$S_q(l) \propto (\varepsilon l)^{\frac{q}{3}}, \quad \delta u(l) \propto l^{\frac{1}{3}}$$

Corrections to Kolmogorov scaling at the observation scale l should depend on l/l_0 and l/L

Quantum field theory approach to turbulence

Quantum field theory approach to hydrodynamic turbulence, viz, a random process described by stochastic equation of the form $\hat{V}[u(x)] = f(x,\cdot)$, where $\hat{V}$ is some nonlinear operator and $f(x,\cdot)$ is Gaussian random force, consists in constructing generation functional $G[A_u(x)]$ such that statistical momenta of the solutions $u(x,\cdot)$ can be obtained by functional differentiation:

$$\langle u(x_1)\ldots u(x_n)\rangle = \left.\frac{\delta^n G[A_u(x)]}{\delta A_u(x_1)\ldots\delta A_u(x_n)}\right|_{A_u=0}$$

The generating functional has the form

$$G[A_u] = \int e^{\int A_u(x)u(x)dx} P[u(x)]\mathcal{D}u,$$

where the probability of particular field configuration $u(x)$ is determined by the equations of motion and the probability of random force configurations:

$$P[u(x)] \sim \int \delta(\hat{V}[u(x)] - f(x))\rho[f]\mathcal{D}f, \qquad \rho[f] \sim e^{-\frac{fD^{-1}f}{2}}$$

Field Doubling Formalism

P.C.Martin, E.D. Sigia, and H.A. Rose, *Phys. Rev. A* **8**(1973)423

The delta function of the equations of motion can be made into the exponent by integration over a imaginary auxiliary field u': $\delta(\cdot) \sim \int \mathcal{D}u'(x) \exp\left(\int dx u'(x)(\cdot)\right)$ .. For the case $\hat{V}[u] =$ "Navier-Stokes equation" this results in quantum field theory generating functional:

$$G[A] = \int \exp\left(S[\Phi] + \int d^d\mathbf{x} dt A\Phi\right) \mathcal{D}\Phi \equiv e^{W[A]},$$

where the field $\Phi = (u, u')$ is the doublet of velocity field $u(t, \mathbf{x})$, and the Martin-Sigia-Rose auxiliary field $u'(t, \mathbf{x})$. The argument $A(t, \mathbf{x}) \equiv (A_u, A_{u'})$ is arbitrary functional source. "Action" itself takes the form

$$S[\Phi] = \frac{1}{2} \int u' D u' + \int u'[-\partial_t u + \nu \Delta u - (u \cdot \partial) u],$$

where $D(x - x') = \langle f(x) f(x') \rangle$. The pressure term is eliminated from the action assuming all vector fields transversal. Functional determinant from $\delta(\cdot)$ is dropped due to redefinition of the Green functions at discontinuity L.Ts.Adzhemyan, A.N. Vasil'ev and Yu.M. Pis'mak, *Teor. Mat. Phys.* **57**(1983) pp.1131-1143(en), 268-281(ru)

Choice of random force

According to Kolmogorov theory the stirring force correlator $D(x-x') = \langle f(x)f(x')\rangle$ should be choosen so that the work performed by random force should be equal to energy dissipation $\langle uf\rangle = \varepsilon$. Besides, the stirring force is assumed to be uncorrelated in time, and be concentrated at large IR scales.

$$\langle \dot{u}_i(t,\mathbf{x})u_i(t,\mathbf{x})\rangle = \int e^{\imath\mathbf{x}(\mathbf{k}_1+\mathbf{k}_2)}\langle f_i(t,\mathbf{k}_1)\int^t f_i(\tau,\mathbf{k}_2)d\tau\rangle \frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}$$

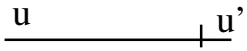
If the stirring force is δ-correlated in time:
$\langle f_i(x)f_j(x')\rangle = \int \frac{d^4k}{(2\pi)^4}d(k)P_{ij}(\mathbf{k})e^{\imath k(x-x')}$, with $P_{ij}(\mathbf{k}) = \delta_{ij} - \frac{k_ik_j}{\mathbf{k}^2}$, we get

$$\varepsilon = \int \frac{d^3\mathbf{k}}{(2\pi)^3}d(k),$$

$G_0(0,\mathbf{x}) = \frac{1}{2}$ was used
Typically this may be $\langle f(k)f(k')\rangle \sim \delta(\omega+\omega')\delta(\mathbf{k}+\mathbf{k}')k^{-y}$ with $y > -2$ for strongly nonequilibrial flows V.Yakhot, S.Orszag. *J.Sci.Comp.* **1**(1986)3

Feynman diagram technique

Action S is a sum of "unperturbed action"



$$S_0[\Phi] = \frac{\Phi K \Phi}{2}, K = \begin{pmatrix} 0 & \partial_t + \hat{L} \\ -\partial_t + \hat{L} & D \end{pmatrix}, \quad \hat{L} = \nu\Delta,$$

$$\langle uu' \rangle = \frac{1}{-\imath\omega + \nu_0 k^2}$$

and the interaction term

$$V[\Phi] = -\frac{1}{2} u_i{}'[\delta_{is}\nabla_j + \delta_{ij}\nabla_s]u_j u_s$$

$$\langle uu \rangle = \frac{d(k)}{\omega^2 + \nu_0^2 k^4}$$

The formal functional integration over Φ performed with the free action S_0 gives the matrix of second moments

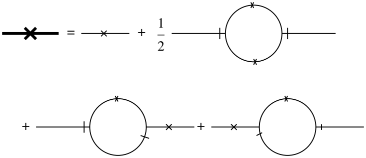
$$V_{ijs} = \frac{\imath}{2}(k_j\delta_{is} + k_s\delta_{ij})$$

$$W[J] = \frac{JK^{-1}J}{2}, K^{-1} = \begin{pmatrix} \frac{D}{|\partial_t - \hat{L}|^2} & (\partial_t - \hat{L})^{-1} \\ (\partial_t - \hat{L})^{-1T} & 0 \end{pmatrix}.$$

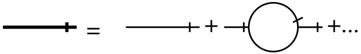
where k is momentum incident to u'.

Achievements of quantum field theory approach to turbulence

- Prove of the Kolmogorov spectrum $E(k) = C_K \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$
- Evaluation of the Kolmogorov constant $C_K \approx 1.6$
- Skewness $S = -\frac{\langle (\nabla u)^3 \rangle}{\langle (\nabla u)^2 \rangle^{\frac{3}{2}}} \approx 0.6$
- Turbulent viscosity
- Passive scalar advecture
- Magnetic hydrodynamics



Evaluation of the pair correlator of velocity field



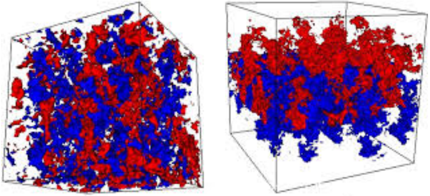
Evaluation of Green function gives corrections to viscosity

Wavelet methods in turbulence

$$u_a(b) = \int \frac{1}{a} \bar{g} \left(\frac{x-b}{a} \right) u(x) dx$$

There are a number of reasons for using wavelets in turbulence

- Intermittency
- Self-similarity
- Fractal structure



(a) (b)

Pictures from asme.org

... without saying a word on wavelet numeric simulations.

- V.Zimin, *Izv. Atmos. Ocean. Phys.* **17**(1981)941
- M.Vergassola and U.Frisch, *Physica D* **54**(1991)58
- J.F.Muzy, E.Bacry and A.Arneodo, *Phys. Rev. Lett* **67**(1991)3515
- C.Meneveau, *J.Fluid.Mech* **232**(1991)469
- M.Farge, *Ann. rev. fluid. mech.* **24**(1992) 395
- N.Astafieva, *Phys. Usp.* **39**(1996)1085
- and many others ...

Multiscale theory of turbulence in wavelet basis

M.Altaisky, *Doklady Physics* **51** (2006)481; with some unjustified assumptions

$\lambda \ll$	$l_0 \ll$	$a < L$
mean free path	Kolmogorov scale	External scale

Energy comes from external scale L and sinks at dissipative scale l_0. To analyze *local* properties of turbulent velocity field $u(t,\mathbf{x})$ we apply continuous wavelet transform (CWT):

$$u_a(\mathbf{b}) = \int_{\mathbb{R}^d} \frac{1}{a^d}\bar{g}\left(\frac{\mathbf{x}-\mathbf{b}}{a}\right) u(\mathbf{x}) d^d\mathbf{x}$$

The reconstruction is:

$$u(\mathbf{x}) = \frac{1}{C_g}\int_0^\infty \frac{da}{a}\int_{\mathbb{R}^d} \frac{1}{a^d} g\left(\frac{\mathbf{x}-\mathbf{b}}{a}\right) u_a(\mathbf{b}) d^d\mathbf{b},$$

with rather loose restrictions on the basic wavelet g: $C_g = \int_0^\infty |\tilde{g}(a)|^2 \frac{da}{a} < \infty$. Technically, CWT is just a filtering of turbulent signal:

$$\tilde{u}_a(\mathbf{k}) = \bar{\tilde{g}}(a\mathbf{k})\tilde{u}(\mathbf{k})$$

Multiscale generating functional

Multiscale quantum field theory:

- $\Phi(x)$ is expressed in terms of $\Phi_a(x)$ using inverse wavelet transform.
- The integration measure $dx \equiv dtd^d\mathbf{x}$ is changed into $d\mu_a = dt\frac{d^d\mathbf{x}da}{a}$

This leads to scale-dependent generating functional

$$G[A] = e^{W[A]} = \int \mathcal{D}\Phi_a(x) e^{S[\Phi_a] + \int \frac{dxda}{a} A_a(x)\Phi_a(x)}$$

All functional derivatives are taken with respect to scale-dependent measure $d\mu_a$:

$$\langle \Phi_{a_1}(x_1)\ldots\Phi_{a_n}(x_n)\rangle_c = \left.\frac{\delta^n W[A]}{\delta A_{a_1}(x_1)\ldots\delta A_{a_n}(x_n)}\right|_{A=0}.$$

Stirring force is uncorrelated in time and scale. It is essentially concentrated on external scale $D_{aa'}(\mathbf{k}) \sim \delta(t-t')\delta(a-a')\delta(a-L)$:

$$\langle \tilde{f}_{ai}(t,\mathbf{k})\tilde{f}_{a'j}(t',\mathbf{k}')\rangle = \delta(t-t')P_{ij}(\mathbf{k})g_0\nu_0^3 C_g(2\pi)^d\delta^d(\mathbf{k}+\mathbf{k}')a\delta(a-a')\delta(a-L)$$

Feynman diagrams in multiscale theory

- Each external line is labeled by a pair (a, k) and a vector index (i).
- Integration in each internal line is performed over $\frac{d\omega}{2\pi}\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{da}{a}\frac{1}{C_g}$
- There are two type of lines: a) Green functions $\langle uu' \rangle$ and b) correlation functions $\langle uu \rangle$; These Green and correlation functions are given by propagator matrix multiplied by wavelet factors $\tilde{g}(a\mathbf{k})$ on each leg.
- Each line with momentum k is proportional to orthogonal projector $P_{ij}(\mathbf{k})$, where i and j are vector indices of the line, i.e.

$$G^{(0)}_{i\alpha,j\beta}(k) = \frac{\tilde{g}(\alpha\mathbf{k})P_{ij}(\mathbf{k})\bar{\tilde{g}}(\beta\mathbf{k})}{-\imath\omega + \nu_0\mathbf{k}^2}, \quad D^{(0)}_{i\alpha,j\beta}(k) = P_{ij}(\mathbf{k})\frac{g_0\nu_0^3}{C_g L}\frac{\tilde{g}(\alpha\mathbf{k})|\tilde{g}(kL)|^2\bar{\tilde{g}}(\beta\mathbf{k})}{|-\imath\omega + \nu_0\mathbf{k}^2|^2}$$

- Each vertex of the diagram is given by $m_{abc}(k) = \frac{\imath}{2}\left(k_b\delta_{ac} + k_c\delta_{ab}\right)$, multiplied by 3 wavelet factors of adjusted lines.
- Statistical momenta of the turbulent velocity field are determined by direct energy cascade. *This means it should be no scales a_i in internal lines less than minimal scale $A = \min_e a_e$ of all external lines.*
- Integration over the propagator lines G over the scales (A, ∞) produce the factor $f_g^2(x)$ where $f_g(x) = \frac{1}{C_g}\int_x^\infty \frac{|\tilde{g}(a)|^2}{a}da, \quad x = kA$

One-loop corrections to viscosity

The corrections to viscosity are determined by the vertex function $\Gamma^{(2)}$, which is inverse to two-point Green function $\Gamma^{(2)}G^{(2)} = 1$.
In null order approximation

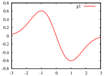
$$\Gamma^{(2)}_0 = \tilde{g}(\alpha\mathbf{k})P_{ij}(\mathbf{k})\bar{\tilde{g}}(\beta\mathbf{k})(-\imath\omega + \nu_0\mathbf{k}^2)$$

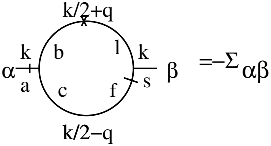
Turbulent contribution to viscosity:
$\Gamma^{(2)} = \Gamma^{(2)}_0 + \Sigma$

$\xi = A/L$

Calculations with g_1 wavelet

$$\tilde{g}_1(k) = -\imath k e^{-\frac{k^2}{2}},\ C_{g_1} = \frac{1}{2},\ f_{g_1}(x) = e^{-x^2}$$



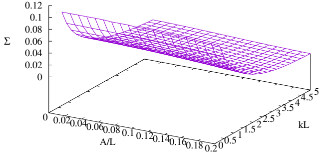


$$\Sigma = -\nu_0 g_0 kL \int_0^\infty \frac{y^2dy}{(2\pi)^2} e^{-(kL)^2(1+4\xi^2)\left(\frac{1}{4}+y^2\right)} \int_0^\pi d\theta \sin\theta \frac{L_{as}(k,p^+,p^-)e^{-(kL)^2 y\cos\theta}}{\frac{1}{4}+y^2-\imath\frac{\omega}{2\nu_0\mathbf{k}^2}}$$

$$L_{as}(k,p^+,p^-) = \frac{\delta_{as}}{4}\left[\frac{(p^+k)(p^+p^-)}{p^{+2}} - (kp^-)\right] + \frac{p_a^+p_s^-}{2}\left[\frac{(p^+k)}{p^{+2}} - \frac{(p^-k)(p^+p^-)}{p^{-2}p^{+2}}\right]$$
$$+ \quad p_a^-p_s^-\left[\frac{(kp^-)}{p^{-2}} - \frac{(p^+k)(p^+p^-)}{2p^{-2}p^{+2}}\right] - \frac{k_ap_s^-}{2} + p_a^+k_s\frac{p^+p^-}{4p^{+2}} - \frac{p_a^-k_s}{4}$$

Renormalization of viscosity: $\nu(\xi) = \nu_L \left(1 - g_L \Sigma^\delta(\xi)\right)$

We can express the "self-energy" contribution to $\Gamma^{(2)}$ as a sum of transversal and longitudinal parts, and treat the transversal part as the effect of turbulent pulsations on viscosity:



$$\Sigma_{as}^{\alpha=\beta=A} = \nu_0 g_0 \Sigma^\delta k^2 \left(\delta_{as} - \frac{k_a k_s}{k^2}\right) + \nu_0 g_0 \Sigma^\lambda k_a k_s,$$

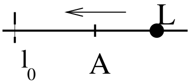
$$\Sigma^\delta = \frac{kL}{128 C_g} \int_0^\infty \frac{y^2 dy}{(2\pi)^2} \frac{e^{-(kL)^2(1+4\xi^2)\left(\frac{1}{4}+y^2\right)}}{\frac{1}{4} + y^2 - \imath \frac{\omega}{2\nu_0 \mathbf{k}^2}}$$

$$\times \int_{-1}^{1} d\mu \frac{(1-\mu^2)(8\mu^2 y^2 + \mu(8y^3 - 10y) + 4y^2 + 1)}{\left(\frac{1}{4y} + y - \mu\right)\left(\frac{1}{4y} + y + \mu\right)} e^{-(kL)^2 y\mu}$$

The contribution $\Sigma^\delta(\xi)$ is finite by and could be used for evaluation of viscosity at smaller scales $\frac{A}{L} \equiv \xi \lesssim 1$, if we know the viscosity ν_L and g_L at external scale L. Neither ν_L nor g_L are known, and we have to iterate the procedure to sequentially smaller scales, same way, but reverse direction, as dynamical renormalization group

Renormalization group equations

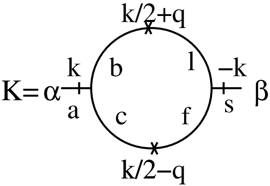
Grid of scales $\frac{l_0}{L} = \xi_0 < \ldots < \xi_{L-2} < \xi_{L-1} < \xi_L = 1$
where $\xi_k = \xi_0 \delta^k, \delta > 1$, $\ln \xi_k = \ln \xi_0 + k \ln \delta$
Viscosity iteration procedure can be written as



$$\frac{\nu_{k-1} - \nu_k}{\nu_k} = -g(\xi_k)\Sigma(\xi_{k-1}) \leftrightarrow \frac{\Delta \ln \nu}{\Delta k} = g(\xi_k)\Sigma(\xi_{k-1})$$

or $\frac{d \ln \nu}{d \ln \xi} = g(\xi) \frac{\Sigma(\xi)}{\ln \delta}$
Force renormalization $D(\xi) = g(\xi)\nu^3(\xi)/L$:

$$\frac{D_{L-1} - D_L}{D_L} = D_L * OneLoopK(\xi_{L-1}).$$



$$\frac{d \ln \nu}{d \ln \xi} = g(\xi)\frac{\Sigma(\xi)}{\ln \delta}, \qquad \frac{d \ln D}{d \ln \xi} = -\frac{K(\xi)}{\ln \delta}$$

$$K(\xi) = \frac{(kL)^3}{16} \int \frac{y^4 dy}{(2\pi)^2} d\mu \frac{e^{-2(Lk)^2(1+2\xi^2)\left(\frac{1}{4}+y^2\right)} \left(\frac{1}{4}+y^2\right)}{\left(\frac{1}{4}+y^2+\imath\frac{2k_0}{\nu_0 \mathbf{k}^2}\right)\left(\frac{1}{4}+y^2-\imath\frac{2k_0}{\nu_0 \mathbf{k}^2}\right)} \frac{(1-\mu^2)(8y^2\mu^2+4y^2+1)}{\left(\frac{1}{4}+y^2-y\mu\right)\left(\frac{1}{4}+y^2+y\mu\right)}$$

Solution of RG equation $\frac{d\ln\nu}{d\ln\xi} = g(\xi)\frac{\Sigma(\xi)}{\ln\delta}$

Since $D(\xi) = g(\xi)\nu^3(\xi)/L$ and $\ln g(\xi) = \ln D + \ln L - 3\ln\nu(\xi)$. We get an equation for running coupling constant $g(\xi)$

$$\frac{d\ln g}{d\ln\xi} = -\frac{K(\xi)}{\ln\delta} - 3g(\xi)\frac{\Sigma(\xi)}{\ln\delta}$$

which has the solution $g(\xi) = \frac{g_L e^{\int_\xi^1 \frac{d\eta}{\eta}K(\eta)}}{1-3g_L\int_\xi^1 \frac{d\xi'}{\xi'}\Sigma(\xi')e^{\int_{\xi'}^1 \frac{d\eta}{\eta}K(\eta)}}$. Substituting this into former equation for viscosity we get the wanted solution.

In practice, since $K(\xi) \ll 1$ and $K(\xi) \ll \Sigma(\xi)$ we can treat the effective force correlator as utmost constant, and hence

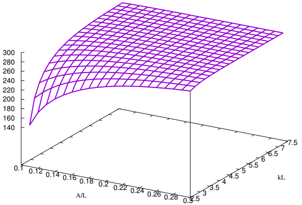
$$g(\xi) = \frac{g_L}{1 - 3g_L\int_\xi^1 \frac{\Sigma(\eta)}{\ln\delta}\frac{d\eta}{\eta}}, \quad \ln\frac{\nu(\xi)}{\nu_L} = -\int_\xi^1 \frac{g_L}{1 - 3g_L\int_{\xi'}^1 \frac{\Sigma(\eta)}{\ln\delta}\frac{d\eta}{\eta}}\frac{\Sigma(\xi')}{\ln\delta}\frac{d\xi'}{\xi'},$$

with $g_0\nu_0^3 \approx g_L\nu_L^3$.

Viscosity dependence on scale

The bare coupling constant g_0 can be found from the equality of energy injection to dissipation ε:

$$\langle \dot{u}_i(t,\mathbf{x}) u_i(t,\mathbf{x})\rangle = \frac{1}{C_g^2}\int e^{\imath \mathbf{x}(\mathbf{k}_1+\mathbf{k}_2)}\tilde{g}(a_1\mathbf{k}_1)\tilde{g}(a_2\mathbf{k}_2)\times$$
$$\times\langle f_{ia_1}(t,\mathbf{k}_1)\int^t f_{ia_2}(\tau,\mathbf{k}_2)d\tau\rangle \frac{d^d\mathbf{k}_1}{(2\pi)^d}\frac{da_1}{a_1}\frac{d^d\mathbf{k}_2}{(2\pi)^d}\frac{da_2}{a_2}$$
$$= \frac{g_0\nu_0^3}{L^{d+1}}\frac{(d-1)}{2C_g}\int_0^\infty |\tilde{g}(\mathbf{y})|^2\frac{d^d\mathbf{y}}{(2\pi)^d},$$



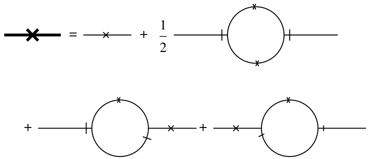
for g_1 wavelet this gives $\varepsilon = \frac{g_0\nu_0^3}{L^4}\frac{3}{8\pi^{3/2}}$ and hence

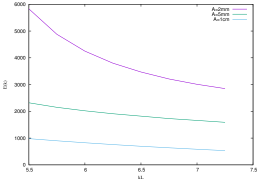
$$g_L = \frac{g_0}{1+3g_0\int_{\xi_0}^1 \Sigma(\xi)\frac{d\xi}{\xi}}, \quad \nu_L = \nu_0\left(\frac{g_0}{g_L}\right)^{\frac{1}{3}}$$

Atmospheric turbulence data: $\varepsilon = 4685 cm^2/s^3$, $l_0 = 0.029 cm$ $x_L = 2\pi$

Calculation of energy spectrum

Pair correlator of the velocity field $\langle \tilde{\mathbf{u}}_a(\omega, \mathbf{k}) \tilde{\mathbf{u}}'_{a'}(\omega', \mathbf{k}') \rangle$ in one-loop approximation is given by a sum of the diagrams shown below





$$
\begin{aligned}
C(k,\xi) = \quad & \frac{g_0 \nu_0^3}{\nu_A(k)} L e^{-(Lk)^2} + \frac{(g_0\nu_0^3)^2}{128} \frac{(Lk)L}{\nu_A^4(k)} e^{-2\xi^2 (Lk)^2} \int_0^\infty \frac{y^2 dy}{(2\pi)^2} \frac{e^{-2(kL)^2(1+2\xi^2)\left(\frac{1}{4}+y^2\right)}}{1+\frac{1}{2}\left(\frac{1}{4}+y^2\right)} \\
\times \quad & \int_{-1}^{1} d\mu \frac{(1-\mu^2)(8\mu^2y^2+4y^2+1)}{\left(\frac{1}{4}+y^2-y\mu\right)\left(\frac{1}{4}+y^2+y\mu\right)} + \frac{(g_0\nu_0^3)^2}{32} \frac{(Lk)L}{\nu_A^4(k)} \\
\times \quad & e^{-(Lk)^2(1+2\xi^2)} \int_0^\infty \frac{y^4 dy}{(2\pi)^2} \frac{e^{-(kL)^2(1+4\xi^2)\left(\frac{1}{4}+y^2\right)}}{1+2\left(\frac{1}{4}+y^2\right)} \\
\times \quad & \int_{-1}^{1} d\mu \frac{(\mu^2-1)(8\mu^2y^2+2\mu y(4y^2-5)+4y^2+1)}{\left(\frac{1}{4}+y^2-y\mu\right)\left(\frac{1}{4}+y^2+y\mu\right)} e^{-(kL)^2 y\mu}
\end{aligned}
$$

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